Welfare effects of Energy Efficiency Policy

Setup

Consider a market for energy-efficient air conditioners where consumers differ in their energy cost savings. Let s represent a consumer's true discounted lifetime energy cost savings from purchasing an efficient AC (relative to an inefficient one), where $s \in [0, \bar{s}]$.

Consumers are uniformly distributed along this savings dimension. We normalize the market size to 1, so $Q \in [0, 1]$ represents the share of consumers purchasing the efficient AC. This will produce graphs that look similar to the one in Allcott and Taubinsky (2015) that we discussed in class.

The **inverse demand function** (willingness to pay as a function of quantity) is:

$$P(Q) = \bar{s}(1 - Q)$$

where $\bar{s}$ is the maximum energy savings (the consumer with highest usage and electricity price).
The efficient AC costs $\Delta c = \$100$ more than the standard model (incremental upfront cost).

Three Demand Functions

1. **Private Rational Demand (Full Information, $\gamma = 1$):** If consumers were fully attentive to energy savings, their inverse demand is:

$$P^{Private}(Q) = 400(1 - Q)$$

2. **Behavioral Demand (Inattention, $\gamma = 0.5$):** Consumers with inattention perceive only $\gamma = 0.5$ of their true energy savings. Their inverse demand is:

$$P^{Behavioral}(Q) = 0.5 \times 400(1 - Q) = 200(1 - Q)$$

This is the demand curve we observe in the market when consumers are inattentive.

3. **Social Demand (Including Externalities):** Energy consumption creates pollution externalities valued at $\phi\bar{e} = \$100$ per efficient AC. The social inverse demand is:

$$P^{Social}(Q) = (400 + 100)(1 - Q) = 500(1 - Q)$$

Market Equilibrium with Inattention (Baseline)

1. Market Quantity with Inattention

   Consumers make decisions based on their perceived benefits (behavioral demand). Find the equilibrium quantity $Q^{Behavioral}$ where the behavioral inverse demand equals the incremental cost:

$$P^{Behavioral}(Q^{Behavioral}) = \Delta c = 100$$

   Solve for $Q^{Behavioral}$.

Solution:

$$P^{Behavioral}(Q^{Behavioral}) = 100$$
$$200(1 - Q^{Behavioral}) = 100$$
$$1 - Q^{Behavioral} = \frac{100}{200} = 0.5$$
$$Q^{Behavioral} = 0.5$$

Half of consumers purchase the efficient AC under inattention.

2. Private Consumer Surplus (Baseline)

   Although consumers make decisions based on perceived benefits, their **actual private welfare** is determined by their **true** energy savings.

   The private consumer surplus is the area between the true private demand curve $P^{Private}(Q)$ and the price $\Delta c = 100$, evaluated at the quantity $Q^{Behavioral} = 0.5$ that consumers actually purchase:

$$CS^{Private} = \int_0^{Q^{Behavioral}} [P^{Private}(q) - \Delta c] \, dq$$

   Calculate the private consumer surplus under inattention.

Solution:

$$CS^{Private} = \int_0^{0.5} [400(1 - q) - 100] \, dq$$
$$= \int_0^{0.5} (300 - 400q) \, dq$$
$$= \left[300q - 200q^2\right]_0^{0.5}$$
$$= 300(0.5) - 200(0.5)^2$$
$$= 150 - 200(0.25) = 150 - 50 = 100$$

Private Consumer Surplus = \$100

3. Social Welfare (Baseline)

   Social welfare uses the social demand curve (which includes externalities):

$$W^{Social} = \int_0^{Q^{Behavioral}} \left[P^{Social}(q) - \Delta c\right] dq$$

   Calculate social welfare under inattention.

Solution:

$$\begin{aligned} W^{Social} &= \int_0^{0.5} [500(1-q) - 100]\, dq \\ &= \int_0^{0.5} (400 - 500q)\, dq \\ &= \left[400q - 250q^2\right]_0^{0.5} \\ &= 400(0.5) - 250(0.25) \\ &= 200 - 62.5 = 137.5 \end{aligned}$$

Social Welfare = $137.5

Optimal Subsidy (Correcting Behavioral Bias)

A policymaker wants to help consumers achieve their **privately optimal** choices (ignoring externalities for now). The privately optimal quantity is where true private demand equals cost:

$$P^{Private}(Q^{Private}) = \Delta c$$

4. Privately Optimal Quantity

   Solve for $Q^{Private}$.

Solution:

$$P^{Private}(Q^{Private}) = 100$$

$$400(1 - Q^{Private}) = 100$$

$$1 - Q^{Private} = \frac{100}{400} = 0.25$$

$$Q^{Private} = 0.75$$

Privately Optimal Quantity = 0.75

5. Required Subsidy

To induce behavioral consumers to purchase $Q^{Private}$ units, what per-unit subsidy τ is needed? The subsidy must satisfy:

$$P^{Behavioral}(Q^{Private}) = \Delta c - \tau$$

Solve for τ.

Solution:

$$P^{Behavioral}(0.75) = 100 - \tau$$
$$200(1 - 0.75) = 100 - \tau$$
$$200(0.25) = 100 - \tau$$
$$50 = 100 - \tau$$
$$\tau = 50$$

Required Subsidy = $50 per unit

The subsidy makes the effective price to consumers $50, which induces them to purchase at the privately optimal level.

6. Private Welfare with Subsidy

With the subsidy, consumers purchase $Q^{Private}$ units at an effective price of $\Delta c - \tau$.

Calculate:

(a) Private consumer surplus with subsidy

(b) Government cost

(c) Net private welfare (consumer surplus minus government cost)

(d) Gain in net private welfare compared to baseline

Solution: (a) Private Consumer Surplus:

Consumers face an effective price of $50, so:

$$CS^{Private,subsidy} = \int_0^{0.75} [400(1 - q) - 50]\, dq$$
$$= \int_0^{0.75} (350 - 400q)\, dq$$
$$= \left[350q - 200q^2\right]_0^{0.75}$$
$$= 350(0.75) - 200(0.75)^2$$
$$= 262.5 - 112.5 = 150$$

Consumer Surplus = $150

(b) Government Cost:

$$G = \tau \times Q^{Private} = 50 \times 0.75 = 37.5$$

(c) Net Private Welfare:

$$W^{Private,net} = CS - G = 150 - 37.5 = 112.5$$

(d) Gain over Baseline:

$$\Delta W = 112.5 - 100 = 12.5$$

The subsidy increases net private welfare by \$12.5, representing the welfare gain from correcting the behavioral bias.

7. Social Welfare with Subsidy

Social welfare with the subsidy includes the external benefits:

$$W^{Social,subsidy} = \int_0^{Q^{Private}} \left[P^{Social}(q) - \Delta c\right] dq$$

Calculate:

(a) Social welfare with subsidy (gross of government cost)

(b) Gain in social welfare compared to baseline

Note: When calculating social welfare, we compute the gross social surplus. The government cost represents a transfer from taxpayers to consumers, not a resource cost to society.

Solution: (a) Social Welfare:

$$\begin{aligned} W^{Social,subsidy} &= \int_0^{0.75} [500(1-q) - 100]\, dq \\ &= \int_0^{0.75} (400 - 500q)\, dq \\ &= \left[400q - 250q^2\right]_0^{0.75} \\ &= 400(0.75) - 250(0.5625) \\ &= 300 - 140.625 = 159.375 \end{aligned}$$

Social Welfare = \$159.375

(b) Gain over Baseline:

$$\Delta W^{Social} = 159.375 - 137.5 = 21.875$$

Social welfare increases by \$21.875. The subsidy corrects the behavioral bias, moving quantity from 0.5 to 0.75, closer to the social optimum.

Mandate (Forcing $Q = 1$)

Now consider a mandate that requires **all** consumers ($Q = 1$) to purchase the efficient AC.

8. Private Welfare with Mandate

   Under a mandate, all consumers purchase the efficient AC. Private consumer surplus is:

$$CS^{Private,mandate} = \int_0^1 [P^{Private}(q) - \Delta c] \, dq$$

   Calculate private consumer surplus under the mandate.

Solution:

$$CS^{Private,mandate} = \int_0^1 [400(1-q) - 100] \, dq$$
$$= \int_0^1 (300 - 400q) \, dq$$
$$= \left[300q - 200q^2\right]_0^1$$
$$= 300 - 200 = 100$$

Private Consumer Surplus = $100

This is the same as the baseline, but the distribution differs. The mandate forces consumers with $Q \in [0.75, 1]$ to purchase a product that costs more than their private benefit. These consumers experience negative surplus, which exactly offsets the gains to consumers with $Q \in [0.5, 0.75]$ who now purchase.

9. Social Welfare with Mandate

   Social welfare with the mandate is:

$$W^{Social,mandate} = \int_0^1 [P^{Social}(q) - \Delta c]\, dq$$

   Calculate social welfare under the mandate.

Solution:

$$\begin{aligned} W^{Social,mandate} &= \int_0^1 [500(1-q) - 100]\, dq \\ &= \int_0^1 (400 - 500q)\, dq \\ &= \left[400q - 250q^2\right]_0^1 \\ &= 400 - 250 = 150 \end{aligned}$$

Social Welfare = $150

This is higher than the baseline ($137.5) but lower than the subsidy ($159.375).